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Letter to the Editor

Comments on "Parametric analysis and fractal-like basins of attraction by modified interpolated cell mapping"

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Ge and Tsen [1] introduced a method using the modified interpolated cell mapping method (MICM, Ref. [2]) to locate all attractors of a dynamical system in a large region. In addition, fractal-like basins of attraction are determined by MICM. In Section 4 in Ref. [1] MICM is applied to the Duffing system:

$$\ddot{x} + 0.1\dot{x} - x + x^3 = 3.2\cos(\omega t) \tag{1}$$

with $0.473 \leqslant \omega \leqslant 0.482$. The variation of ω is taken as 0.0001. MICM finds two sequences of period doubling solutions when ω is diminished from 0.482. In the limit two strange attractors are obtained for $\omega = 0.4753$ with transition to a large chaotic attractor for $\omega = 0.4736$.

The occurrence of two additional chaotic attractors for $\omega = 0.479$ should be mentioned which apparently has been overlooked by applying MICM. Besides the two periodic attractors, already mentioned in Ref. [1], the following additional chaotic attractors are found. With appropriate initial conditions, the first attractor in the phase plane at multiple values of the forced period $T = 2\pi/\omega$, is located in the region $(1.1,2.3) \times (0.2,2.7)$ and consists of three small clusters of points (Fig. 1(a)). The second attractor (Fig. 1(b)) is also constituted of three small clusters and is situated in $(0.2,1.4) \times (-3.4,0.6)$. The coexistence of the four attractors (two periodic attractors both with the period $2T$ and the two chaotic attractors just mentioned) is illustrated in Fig. 2 where the basins of attraction in the phase plane are shown in the enlarged region $(0,2.7) \times (-3.6,2.7)$. The basins of attraction of the two periodic attractors are represented in light- and dark-grey. The two chaotic attractors are shown by the three clusters of points indicated in black in its relevant basin of attraction represented in white and by the three clusters in white in its corresponding domain in black. The domains of attraction have been constructed with the use of the package DYNAMICS [3].

It has been pointed out that the two chaotic attractors at $\omega = 0.479$ are generated by two periodic solutions at $\omega = 0.4792$ both having the period $3T$. At this value of ω four periodic

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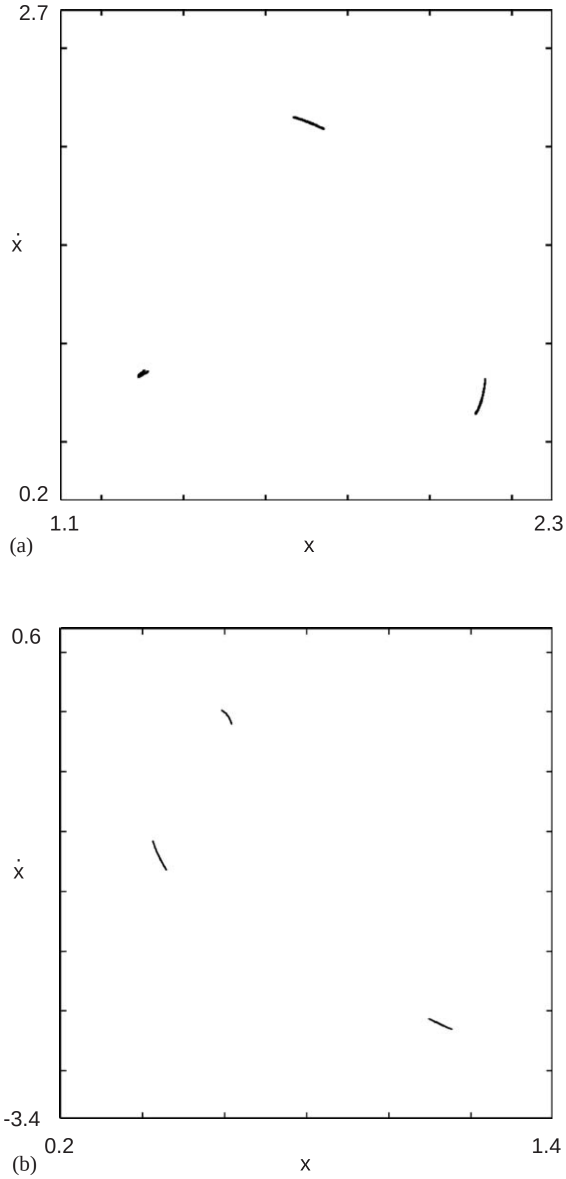

Fig. 1. Chaotic attractors in the phase plane for $\omega = 0.479$: (a) first attractor in the region $(1.1,2.3) \times (0.2,2.7)$; and (b) second attractor in the region $(0.2,1.4) \times (-3.4,0.6)$.

attractors coexist with their Poincaré section points at $t = 0$ in the phase plane given by

period-2: (1.67911, 1.09367), (2.00459, 1.06905);
period-2: (0.56436, −0.40174), (0.67350, −1.40672);
period-3: (1.71312, 2.11566), (2.12409, 0.71385), (1.30740, 0.85696);
period-3: (1.11631, −2.59428), (0.43890, −1.18418), (0.60765, −0.05038).

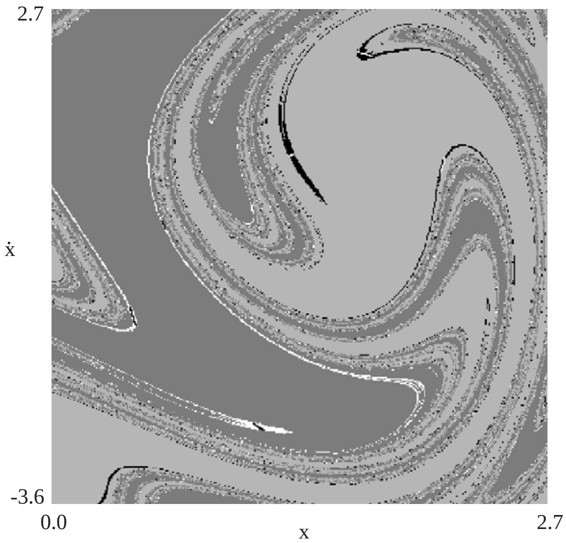

Fig. 2. Basins of attraction in the phase plane for $\omega = 0.479$. Coexistence of four attractors: $2T$-periodic, $2T$-periodic, chaotic and chaotic. Basins are indicated in light-grey, dark-grey, black and white.

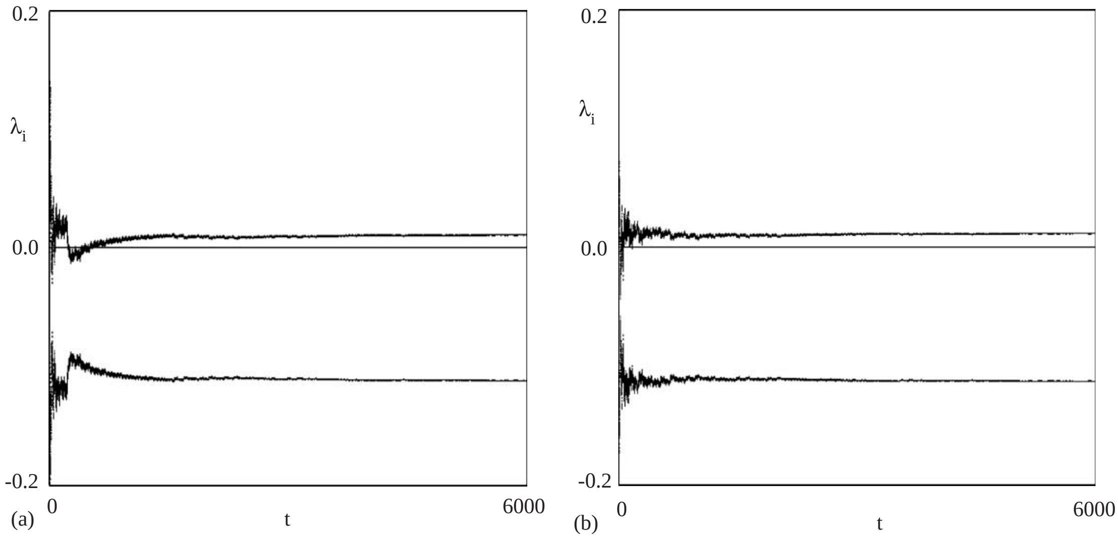

Fig. 3. Behaviour of the Liapounov exponents λ_i over long times for chaotic motion at $\omega = 0.479$: (a) for first attractor; and (b) for second attractor.

One of the most reliable methods for concluding that the behaviour of the system is chaotic, is the numerical computation of the Liapounov exponents [4]. With $\omega = 0.479$ the stabilization of the behaviour of the Liapounov exponents λ_1 and λ_2 (λ_3 is always zero) over long times is illustrated in Fig. 3(a) for the first attractor and in Fig. 3(b) for the second attractor. Since in both cases one of the Liapounov exponents is positive, the corresponding motion of the system is chaotic.

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